

Common Fixed Point Theorem for Two Multi Valued Mappings Satisfying Rational Inequality in Complex Valued Metric Space

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Abstract: In this paper common fixed point theorem has been proved for two multi-valued mappings satisfying a rational inequality in complex valued metric space. Also we extend and strengthened the results given in [5,11].

Keywords: multi-valued mapping, common fixed point, complex valued metric space.

1. INTRODUCTION

The concept of Multi valued contraction mapping was initiated by Nadler[1] and Markin[2]. Results for stability of fixed points for multi valued mappings have been discussed in many authors[3-10]. This paper deals with some common fixed point theorems which are established for multi valued mapping in complex valued metric space with rational inequality in complex valued metric space. Azam et al.(numer.Funct.anal.Optim.33(5):590-600,2012) introduced the notion of complex valued metric space and proved some common fixed point theorems in the context of complex valued metric space, we will use rational inequality for two multi-valued mapping.

Let us recall a natural relation on \mathbb{C} , for $z_1, z_2 \in \mathbb{C}$, define a partial order \preceq on \mathbb{C} as follows;

$$z_1 \preceq z_2 \text{ iff } \operatorname{Re}(z_1) \leq \operatorname{Re}(z_2), \operatorname{Im}(z_1) \leq \operatorname{Im}(z_2)$$

it follows that

$$z_1 \preceq z_2$$

if one of the following conditions is satisfied:

- $\operatorname{Re}(z_1) = \operatorname{Re}(z_2), \operatorname{Im}(z_1) = \operatorname{Im}(z_2)$
- $\operatorname{Re}(z_1) < \operatorname{Re}(z_2), \operatorname{Im}(z_1) = \operatorname{Im}(z_2)$
- $\operatorname{Re}(z_1) = \operatorname{Re}(z_2), \operatorname{Im}(z_1) < \operatorname{Im}(z_2)$
- $\operatorname{Re}(z_1) < \operatorname{Re}(z_2), \operatorname{Im}(z_1) < \operatorname{Im}(z_2)$

In particular, we will write $z_1 \not\preceq z_2$ if $z_1 \neq z_2$ and one of a),b),c),d) is not satisfied and we will write $z_1 < z_2$ if only (d) is satisfied. Note that

$$0 \preceq z_1 \not\preceq z_2 \Leftrightarrow |z_1| < |z_2|,$$

$$z_1 \preceq z_2, z_1 < z_2 \Leftrightarrow z_1 < z_3$$

Definition 1.2 let X be a nonempty set. A mapping $d: X \times X \rightarrow \mathbb{C}$ satisfies the following conditions

(CM1) $0 \preceq d(x,y)$ for all $x,y \in X$ and $d(x,y) = 0 \Leftrightarrow x=y$.

(CM2) $d(x,y) = d(y,x)$ for all $x,y \in X$

(CM3) $d(x,y) \preceq d(x,z)+d(z,y)$ for all $x,y,z \in X$.

Then d is called a complex valued metric on X and (X,d) is called a complex valued metric space.

It is obvious that this concept is generalization of the classic metric. In fact, if $d: X \times X \rightarrow \mathbb{R}$ satisfies ((CM1)-(CM3)), then this d is a metric in the classical sense, that is, the following conditions are satisfies:

(M1) $0 \leq d(x,y)$ for all $x,y \in X$ and $d(x,y)=0 \Leftrightarrow x=y$.

(M2) $d(x,y)=d(y,x)$ for all $x,y \in X$

(M3) $d(x,y) \leq d(x,z)+d(z,y)$ for all $x,y,z \in X$.

There are so many more different and interesting type of metric spaces and classical theories of metric space for example see[3,4].

Example 1.3. Let $X = \mathbb{C}$. Define the mapping $d : X \times X \rightarrow \mathbb{C}$ by

$$d(z_1, z_2) = e^{ai} |z_1 - z_2|, \text{ for all } z_1, z_2 \in X.$$

Then (X,d) is a complex valued metric space.

Definitions 1.4. Let \mathbb{C} be a complex valued metric space,

- We say that a sequence $\{x_n\}$ is said to be a Cauchy sequence be a sequence in $x \in X$ If for every $\varepsilon \in \mathbb{C}$, with $0 < \varepsilon$ there is $n_0 \in \mathbb{N}$ such that for all $n > n_0$ such that $d(x_n, x_m) < \varepsilon$.
- We say that a sequence $\{x_n\}$ converges to an element x if for every $\varepsilon \in \mathbb{C}$, with $0 < \varepsilon$ there exist an integer $n_0 \in \mathbb{N}$ such that for all $n > n_0$ such that $d(x_n, x) < \varepsilon$ and we write $x_n \xrightarrow{d} x$.
- We say that (x,d) is complete if every Cauchy sequence in X converges to a point in X .

1.1 Main Result:

Let (X,d) be a complex valued metric space.

Let family of non-empty, closed and bounded subsets of a complex valued metric space is denoted by $CB(X)$.

we denote $s(z_1) = \{z_2 \in \mathbb{C} : z_1 \preceq z_2\}$ for $z_1 \in \mathbb{C}$, and $s(a,b) = \bigcup_{b \in B} s(d(a,b)) = \bigcup_{b \in B} \{z \in \mathbb{C} : d(a,b) \preceq z\}$ for $a \in X$ and $B \in CB(X)$.

For $A, B \in CB(X)$, we denote

$$s(a,b) = (\bigcup_{a \in A} s(a,B)) \cap (\bigcup_{b \in B} s(b,A)).$$

Common fixed result discussed by Khan [3] can be obtained in the setting of complex valued metric space.

Theorem 2.1 let (X,d) be a complete complex valued metric space and let $S, T: X \rightarrow CB(X)$ be multi valued mapping with greatest lower bound property such that,

$$\alpha \frac{[d(x, Sx)d(x, Ty) + d(y, Ty)d(y, Sx)]}{d(x, Ty) + d(y, Sx)} + \beta \frac{[d(x, Tx)d(y, Sy) + d(x, Ty)d(y, Tx)]}{d(y, Sy) + d(Ty, Sy)} + \gamma \frac{[d(x, Sx)d(y, Sy) + d(Sx, Tx)d(Sy, Tx)]}{d(Sx, Ty) + d(y, Tx)} \in s(Sx, Ty)$$

$\forall x, y \in X$ and $0 \leq \alpha + \beta + \gamma < 1$. Then S and T have common fixed point.

Proof Let $x_0 \in X$ and $x_1 \in Sx_0, Tx_0$. from (1.1), we have

$$\alpha \frac{[d(x_0, Sx_0)d(x_0, Tx_1) + d(x_1, Tx_1)d(x_1, Sx_0)]}{d(x_0, Tx_1) + d(x_1, Sx_0)} + \beta \frac{[d(x_0, Tx_0)d(x_0, Sx_1) + d(x_0, Tx_0)d(x_1, Tx_0)]}{d(x_1, Sx_1) + d(Tx_1, Sx_1)} + \gamma \frac{[d(x_0, Sx_0)d(x_1, Sx_1) + d(Sx_0, Tx_0)d(Sx_1, Tx_0)]}{d(Sx_0, Tx_1) + d(x_1, Tx_0)} \in s(Sx_0, Tx_1)$$

$$\alpha \frac{[d(x_0, Sx_0)d(x_0, Tx_1) + d(x_1, Tx_1)d(x_1, Sx_0)]}{d(x_0, Tx_1) + d(x_1, Sx_0)} + \beta \frac{[d(x_0, Tx_0)d(x_0, Sx_1) + d(x_0, Tx_0)d(x_1, Tx_0)]}{d(x_1, Sx_1) + d(Tx_1, Sx_1)} + \gamma \frac{[d(x_0, Sx_0)d(x_1, Sx_1) + d(Sx_0, Tx_0)d(Sx_1, Tx_0)]}{d(Sx_0, Tx_1) + d(x_1, Tx_0)} \in \bigcap_{x \in Sx_0} s(x, Tx_1)$$

i.e

$$\alpha \frac{[d(x_0, Sx_0)d(x_0, Tx_1) + d(x_1, Tx_1)d(x_1, Sx_0)]}{d(x_0, Tx_1) + d(x_1, Sx_0)} + \beta \frac{[d(x_0, Tx_0)d(x_0, Sx_1) + d(x_0, Tx_0)d(x_1, Tx_0)]}{d(x_1, Sx_1) + d(Tx_1, Sx_1)} + \gamma \frac{[d(x_0, Sx_0)d(x_1, Sx_1) + d(Sx_0, Tx_0)d(Sx_1, Tx_0)]}{d(Sx_0, Tx_1) + d(x_1, Tx_0)} \in s(x, Tx_1) \forall x \in Sx_0$$

since $x_1 = Sx_0$, so we have

$$\alpha \frac{[d(x_0, Sx_0)d(x_0, Tx_1) + d(x_1, Tx_1)d(x_1, Sx_0)]}{d(x_0, Tx_1) + d(x_1, Sx_0)} + \beta \frac{[d(x_0, Tx_0)d(x_0, Sx_1) + d(x_0, Tx_0)d(x_1, Tx_0)]}{d(x_1, Sx_1) + d(Tx_1, Sx_1)} + \gamma \frac{[d(x_0, Sx_0)d(x_1, Sx_1) + d(Sx_0, Tx_0)d(Sx_1, Tx_0)]}{d(Sx_0, Tx_1) + d(x_1, Tx_0)} \in s(x_1, Tx_1)$$

$$\alpha \frac{[d(x_0, Sx_0)d(x_0, Tx_1) + d(x_1, Tx_1)d(x_1, Sx_0)]}{d(x_0, Tx_1) + d(x_1, Sx_0)} + \beta \frac{[d(x_0, Tx_0)d(x_0, Sx_1) + d(x_0, Tx_0)d(x_1, Tx_0)]}{d(x_1, Sx_1) + d(Tx_1, Sx_1)} + \gamma \frac{[d(x_0, Sx_0)d(x_1, Sx_1) + d(Sx_0, Tx_0)d(Sx_1, Tx_0)]}{d(Sx_0, Tx_1) + d(x_1, Tx_0)} \in s(x_1, Tx_1) = \bigcup_{x \in Tx_1} s(d(x_1, x))$$

there exist $x_2 \in Sx_1, Tx_1$ such that,

$$\alpha \frac{[d(x_0, Sx_0)d(x_0, Tx_1) + d(x_1, Tx_1)d(x_1, Sx_0)]}{d(x_0, Tx_1) + d(x_1, Sx_0)} + \beta \frac{[d(x_0, Tx_0)d(x_0, Sx_1) + d(x_0, Tx_0)d(x_1, Tx_0)]}{d(x_1, Sx_1) + d(Tx_1, Sx_1)} + \gamma \frac{[d(x_0, Sx_0)d(x_1, Sx_1) + d(Sx_0, Tx_0)d(Sx_1, Tx_0)]}{d(Sx_0, Tx_1) + d(x_1, Tx_0)} \in s(d(x_1, x_2))$$

i.e,

$$d(x_1, x_2) \leq \alpha \frac{[d(x_0, x_1)d(x_0, x_2) + d(x_1, x_2)d(x_1, x_1)]}{d(x_0, x_2) + d(x_1, x_1)} + \beta \frac{[d(x_0, x_1)d(x_1, x_2) + d(x_1, x_1)d(x_2, x_1)]}{d(x_1, x_2) + d(x_2, x_2)} + \gamma \frac{[d(x_0, x_1)d(x_1, x_2) + d(x_1, x_1)d(x_2, x_1)]}{d(x_1, x_2) + d(x_1, x_1)}$$

By using the greatest lower bound property of s and T , we have

$$d(x_1, x_2) \leq \alpha \frac{[d(x_0, x_1)d(x_0, x_2)]}{d(x_0, x_2)} + \beta \frac{[d(x_0, x_1)d(x_1, x_2)]}{d(x_1, x_2)} + \gamma \frac{[d(x_0, x_1)d(x_1, x_2)]}{d(x_1, x_2)}$$

$$d(x_1, x_2) \leq \alpha \frac{[|d(x_0, x_1)||d(x_0, x_2)|]}{|d(x_0, x_2)|} + \beta \frac{[|d(x_0, x_1)||d(x_1, x_2)|]}{|d(x_1, x_2)|} + \gamma \frac{[|d(x_0, x_1)||d(x_1, x_2)|]}{|d(x_1, x_2)|}$$

$$d(x_1, x_2) \leq \alpha |d(x_0, x_1)| + \beta |d(x_0, x_1)| + \gamma |d(x_0, x_1)|$$

$$d(x_1, x_2) \leq (\alpha + \beta + \gamma) |d(x_0, x_1)|$$

Similarly,

$$d(x_2, x_3) \leq (\alpha + \beta + \gamma) |d(x_1, x_2)|$$

$$\leq (\alpha + \beta + \gamma)^2 |d(x_0, x_1)|$$

$$d(x_3, x_4) \leq (\alpha + \beta + \gamma)^3 |d(x_2, x_3)|$$

$$\leq (\alpha + \beta + \gamma)^4 |d(x_1, x_2)|$$

$$\leq (\alpha + \beta + \gamma)^5 |d(x_0, x_1)|$$

Repeatedly we can construct a sequence $\{x_n\}$ in x such that $n=0,1,2,3,\dots$,

$$|d(x_n, x_m)| \leq |d(x_n, x_{n+1})| + |d(x_{n+1}, x_{n+2})| + \dots + |d(x_{m-1}, x_m)|$$

$$\leq (\alpha + \beta + \gamma)^n |d(x_0, x_1)|$$

With $0 \leq (\alpha + \beta + \gamma)^1 < 1$, $x_{2n+1} \in Sx_{2n}$ and $x_{2n+2} \in Tx_{2n+1}$

For m.n, we have

$$|d(x_n, x_m)| \leq |d(x_n, x_{n+1})| + |d(x_{n+1}, x_{n+2})| + \dots + |d(x_{m-1}, x_m)|$$

$$\leq [(\alpha + \beta + \gamma)^n + (\alpha + \beta + \gamma)^{n+1} + \dots + (\alpha + \beta + \gamma)^{m-1}] |d(x_0, x_1)|$$

And so

$$|d(x_n, x_m)| \leq \left(\frac{(\alpha + \beta + \gamma)^n}{1 - (\alpha + \beta + \gamma)^1} \right) |d(x_0, x_1)|$$

And so

$$|d(x_n, x_m)| \leq \left(\frac{(\alpha + \beta + \gamma)^n}{1 - (\alpha + \beta + \gamma)^1} \right) |d(x_0, x_1)| \rightarrow 0 \text{ as } m, n \rightarrow \infty$$

And hence we have a Cauchy sequence $\{x_n\}$ in X, also X is complete and hence the convergent point will be in X i.e. $\exists v \in X \exists x_n \rightarrow v \text{ as } n \rightarrow \infty$. we now show that $v \in Tv$ and $v \in Sv$. from (1.1)

$$\alpha \frac{[d(x_{2n}, Sx_{2n})d(x_{2n}, Tv) + d(v, Tv)d(v, Sx_{2n})]}{d(x_{2n}, Tv) + d(v, Sx_{2n})} + \beta \frac{[d(x_{2n}, Tx_{2n})d(v, Sv) + d(x_{2n}, Tv)d(v, Tx_{2n})]}{d(v, Sv) + d(Tv, Sv)}$$

$$+ \gamma \frac{[d(x_{2n}, Sx_{2n})d(v, Sv) + d(Sx_{2n}, Tx_{2n})d(Sx_{2n}, Tx_{2n})]}{d(Sx_{2n}, Tv) + d(v, Tx_{2n})} \in s(Sx_{2n}, Tv)$$

$$\alpha \frac{[d(x_{2n}, Sx_{2n})d(x_{2n}, Tv) + d(v, Tv)d(v, Sx_{2n})]}{d(x_{2n}, Tv) + d(v, Sx_{2n})} + \beta \frac{[d(x_{2n}, Tx_{2n})d(v, Sv) + d(x_{2n}, Tv)d(v, Tx_{2n})]}{d(v, Sv) + d(Tv, Sv)}$$

$$+ \gamma \frac{[d(x_{2n}, Sx_{2n})d(v, Sv) + d(Sx_{2n}, Tx_{2n})d(Sx_{2n}, Tx_{2n})]}{d(Sx_{2n}, Tv) + d(v, Tx_{2n})} \in \left(\bigcap_{s \in Sx_{2n}} s(x, Tv) \right)$$

also,

$$\alpha \frac{[d(x_{2n}, Sx_{2n})d(x_{2n}, Tv) + d(v, Tv)d(v, Sx_{2n})]}{d(x_{2n}, Tv) + d(v, Sx_{2n})} + \beta \frac{[d(x_{2n}, Tx_{2n})d(v, Sv) + d(x_{2n}, Tv)d(v, Tx_{2n})]}{d(v, Sv) + d(Tv, Sv)}$$

$$+ \gamma \frac{[d(x_{2n}, Sx_{2n})d(v, Sv) + d(Sx_{2n}, Tx_{2n})d(Sx_{2n}, Tx_{2n})]}{d(Sx_{2n}, Tv) + d(v, Tx_{2n})} \in s(x, Tv) \quad \forall x \in Sx_{2n}$$

since,

$x_{2n+1} \in Sx_{2n}$, so we have

$$\alpha \frac{[d(x_{2n}, Sx_{2n})d(x_{2n}, Tv) + d(v, Tv)d(v, Sx_{2n})]}{d(x_{2n}, Tv) + d(v, Sx_{2n})} + \beta \frac{[d(x_{2n}, Tx_{2n})d(v, Sv) + d(x_{2n}, Tv)d(v, Tx_{2n})]}{d(v, Sv) + d(Tv, Sv)}$$

$$+ \gamma \frac{[d(x_{2n}, Sx_{2n})d(v, Sv) + d(Sx_{2n}, Tx_{2n})d(Sx_{2n}, Tx_{2n})]}{d(Sx_{2n}, Tv) + d(v, Tx_{2n})} \in s(x_{2n+1}, Tv)$$

by definition

$$\alpha \frac{[d(x_{2n}, Sx_{2n})d(x_{2n}, Tv) + d(v, Tv)d(v, Sx_{2n})]}{d(x_{2n}, Tv) + d(v, Sx_{2n})} + \beta \frac{[d(x_{2n}, Tx_{2n})d(v, Sv) + d(x_{2n}, Tv)d(v, Tx_{2n})]}{d(v, Sv) + d(Tv, Sv)}$$

$$+ \gamma \frac{[d(x_{2n}, Sx_{2n})d(v, Sv) + d(Sx_{2n}, Tx_{2n})d(Sx_{2n}, Tx_{2n})]}{d(Sx_{2n}, Tv) + d(v, Tx_{2n})} \in s(x_{2n+1}, Tv) = \left(\bigcap_{u \in Tu} s(d(x_{2n+1}, u)) \right)$$

there exist some $v_n \in Tv$ such that

$$\alpha \frac{[d(x_{2n}, Sx_{2n})d(x_{2n}, Tv) + d(v, Tv)d(v, Sx_{2n})]}{d(x_{2n}, Tv) + d(v, Sx_{2n})} + \beta \frac{[d(x_{2n}, Tx_{2n})d(v, Sv) + d(x_{2n}, Tv)d(v, Tx_{2n})]}{d(v, Sv) + d(Tv, Sv)}$$

$$+ \gamma \frac{[d(x_{2n}, Sx_{2n})d(v, Sv) + d(Sx_{2n}, Tx_{2n})d(Sx_{2n}, Tx_{2n})]}{d(Sx_{2n}, Tv) + d(v, Tx_{2n})} \in s(d(x_{2n+1}, v_n))$$

i.e. ,

$$d(x_{2n+1}, v_n) \leq \alpha \frac{[d(x_{2n}, Sx_{2n})d(x_{2n}, Tv) + d(v, Tv)d(v, Sx_{2n})]}{d(x_{2n}, Tv) + d(v, Sx_{2n})} + \beta \frac{[d(x_{2n}, Tx_{2n})d(v, Sv) + d(x_{2n}, Tv)d(v, Tx_{2n})]}{d(v, Sv) + d(Tv, Sv)} + \gamma \frac{[d(x_{2n}, Sx_{2n})d(v, Sv) + d(Sx_{2n}, Tx_{2n})d(Sx_{2n}, Tx_{2n})]}{d(Sx_{2n}, Tv) + d(v, Tx_{2n})}$$

by using the greatest lower bound property of S and T, we have

$$d(x_{2n+1}, v_n) \leq \alpha \frac{[d(x_{2n}, x_{2n+1})d(x_{2n}, v_n) + d(v, v_n)d(v, x_{2n+1})]}{d(x_{2n}, v_n) + d(v, x_{2n+1})} + \beta \frac{[d(x_{2n}, Tx_{2n})d(v, Sv) + d(x_{2n}, v_n)d(v, Tx_{2n})]}{d(v, Sv) + d(v_n, Sv)} + \gamma \frac{[d(x_{2n}, x_{2n})d(v, Sv) + d(x_{2n+1}, Tx_{2n})d(Sv, Tx_{2n})]}{d(x_{2n+1}, Tv) + d(v, Tx_{2n})}$$

since

$$d(v, v_n) \leq d(v, x_{2n+1}) + d(x_{2n+1}, v_n)$$

$$d(v, v_n)$$

\leq

$$d(v, x_{2n+1}) + \alpha \frac{[d(x_{2n}, x_{2n+1})d(x_{2n}, v_n) + d(v, v_n)d(v, x_{2n+1})]}{d(x_{2n}, v_n) + d(v, x_{2n+1})} + \beta \frac{[d(x_{2n}, Tx_{2n})d(v, Sv) + d(x_{2n}, v_n)d(v, Tx_{2n})]}{d(v, Sv) + d(v_n, Sv)} + \gamma \frac{[d(x_{2n}, x_{2n})d(v, Sv) + d(x_{2n+1}, Tx_{2n})d(Sv, Tx_{2n})]}{d(x_{2n+1}, Tv) + d(v, Tx_{2n})}$$

$$|d(v, v_n)| \leq |d(v, x_{2n+1})|$$

$$+ \alpha \frac{[|d(x_{2n}, x_{2n+1})||d(x_{2n}, v_n)| + |d(v, v_n)||d(v, x_{2n+1})|]}{|d(x_{2n}, v_n)| + |d(v, x_{2n+1})|} + \beta \frac{[|d(x_{2n}, Tx_{2n})||d(v, Sv)| + |d(x_{2n}, v_n)||d(v, Tx_{2n})|]}{|d(v, Sv)| + |d(v_n, Sv)|} + \gamma \frac{[|d(x_{2n}, x_{2n})||d(v, Sv)| + |d(x_{2n+1}, Tx_{2n})||d(Sv, Tx_{2n})|]}{|d(x_{2n+1}, Tv)| + |d(v, Tx_{2n})|}$$

By letting $n \rightarrow \infty$ in above inequality, $|d(v, v_n)| \rightarrow 0$. By the definition of convergence we have $v_n \rightarrow v$ as $n \rightarrow \infty$. since Tv is closed, so $v \in Tv$. similarly, it follows that $v \in Sv$. thus s and T have a common fixed point.

Corollary 2.1:

The above theorem can also be generalized as,

Let (X, d) be a complete complex valued metric space and let $S, T: X \rightarrow CB(X)$ be multi valued mapping with greatest lower bound property such that,

$$\alpha \frac{[d(x, Sx)d(x, Ty) + d(y, Ty)d(y, Sx)]}{d(x, Ty) + d(y, Sx)} + \beta \frac{[d(x, Tx)d(y, Sy) + d(x, Ty)d(y, Tx)]}{d(y, Sy) + d(Ty, Sy)} + \gamma \frac{[d(x, Sx)d(y, Sy) + d(Sx, Ty)d(Sy, Tx)]}{d(Sx, Ty) + d(y, Tx)} + \delta \frac{[d(x, Tx)d(x, Sy) + d(Sy, Ty)d(Tx, Sy)]}{d(x, Ty) + d(Sx, Tx)} + \varepsilon \frac{[d(x, y)d(Sx, Sy) + d(Ty, Sy)d(y, Sx)]}{d(Tx, Sy) + d(y, Sx)} \in s(Sx, Ty)$$

$0 \leq \alpha + \beta + \gamma + \delta + \varepsilon < 1$. Then S and T have a common fixed point.

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